

# Keysight Technologies

## Understanding Reading Accuracy and Resolution in a 6 1/2-Digit DMM

Application Note



# Introduction

This application note explains the reality and the mathematics behind digital multimeter (DMM) resolution and DC noise specifications, which directly correlate to the number of digits available from the DMM at a

given resolution and speed. In this paper, we use the 6<sup>1</sup>/<sub>2</sub>-digit Keysight Technologies, Inc. 34410A DMM as an example.

## DMM accuracy and resolution

Figure 1 shows the DC voltage accuracy specifications for the 34410A DMM. On the 10 V range, and with the integration time set to 100 PLC (“power line cycles” – 1 PLC is 16.667 ms

## DC Characteristics

Accuracy Specifications (% of reading + % of range)<sup>1</sup>

Function Coefficient	Range <sup>3</sup>	Test Current or Burden Voltage	24 Hour <sup>2</sup> T <sub>CAL</sub> ±1°C	90 Day T <sub>CAL</sub> ±5°C	1 Year T <sub>CAL</sub> ±5°C	Temperature 0°C to (T <sub>CAL</sub> -5°C) (T <sub>CAL</sub> +5°C) to 55°C
DC	100.0000 mV		0.0030+0.0030	0.0040+0.0035	0.0050+0.0035	0.0005+0.0005
	1.000000 V		0.0020+0.0006	0.0030+0.0007	0.0035+0.0007	0.0005+0.0001
	10.00000 V		0.0015+0.0004	0.0020+0.0005	0.0030+0.0005	0.0005+0.0001
	100.0000 V		0.0020+0.0006	0.0035+0.0006	0.0040+0.0006	0.0005+0.0001
	1000.000 V <sup>5</sup>		0.0020+0.0006	0.0035+0.0006	0.0040+0.0006	0.0005+0.0001
Resistance <sup>4</sup>	100.0000 Ω	1 mA current source	0.0030+0.0030	0.008+0.004	0.010+0.004	0.0006+0.0005
	1.000000 kΩ	1 mA	0.0020+0.0005	0.007+0.001	0.010+0.001	0.0006+0.0001
	10.00000 kΩ	100 μA	0.0020+0.0005	0.007+0.001	0.010+0.001	0.0006+0.0001
	100.0000 kΩ	10 μA	0.0020+0.0005	0.007+0.001	0.010+0.001	0.0006+0.0001
	1.000000 MΩ	5.0 μA	0.0020+0.0010	0.010+0.001	0.012+0.001	0.0010+0.0002
	10.00000 MΩ	500 nA	0.0100+0.0010	0.030+0.001	0.040+0.001	0.0030+0.0004
	100.0000 MΩ	500 nA    10 MΩ	0.200+0.001	0.600+0.001	0.800+0.001	0.1000+0.0001
	1000.000 MΩ	500 nA    10 MΩ	2.000+0.001	6.000+0.001	8.000+0.001	1.0000+0.0001
DC Current	100.0000 μA	<0.03 V burden V	0.010+0.020	0.040+0.025	0.050+0.025	0.0020+0.0030
	1.000000 mA	<0.3 V	0.007+0.006	0.030+0.006	0.050+0.006	0.0020+0.0005
	10.00000 mA	<0.03 V	0.007+0.020	0.030+0.020	0.050+0.020	0.0020+0.0020
	100.0000 mA	<0.3 V	0.010+0.004	0.050+0.005	0.050+0.005	0.0020+0.0005
	1.000000 A	<0.80 V	0.050+0.006	0.080+0.010	0.100+0.010	0.0050+0.0010
	3.00000 A	<2.0 V	0.100+0.020	0.120+0.020	0.150+0.020	0.0050+0.0020
Continuity	1000 Ω	1 mA test current	0.002+0.010	0.008+0.020	0.010+0.020	0.0010+0.0020
Diode Test	1.0000 V <sup>6</sup>	1 mA test current	0.002+0.010	0.008+0.020	0.010+0.020	0.0010+0.0020

1. Specifications are for 90-minute warm-up and integration setting of 100 NPLC. For <100 NPLC add the appropriate "RMS Noise Adder."
2. Relative to calibration standards
3. 20% overrange on all ranges except 1000 VDC, 3 A range
4. Specifications are for 4-wire ohms function, or 2-wire ohms using Math Null. Without Math Null, add 0.2 Ω additional error in 2-wire ohms function.
5. For each additional volt over ±500 VDC, add 0.02 mV of error.
6. Accuracy specifications are for the voltage measured at the input terminals only. 1 mA test current is typical. Variation in the current source will create some variation in the voltage drop across a diode junction.

Figure 1. DC accuracy specifications for the Keysight 34410A DMM

## Performance Versus Integration Time

60 Hz (50 Hz) power line frequency

Integration Time Number of Power Line Cycles (NPLC)	Resolution ppm Range <sup>1</sup>	NMR dB <sup>2</sup>	Readings per Second <sup>3</sup>	RMS Noise Adder % Range <sup>4</sup>		
				DCV 10, 1000 V	DCV 1, 100 V Resistance 1k, 10k Ohm	DCV 0.1 V Resistance 100 Ohm DCI 1 Amp
0.001 <sup>6</sup>	30	0	50,000	0.0060	0.0100	0.1000
0.002 <sup>6</sup>	15	0	25,000	0.0030	0.0060	0.0600
0.006	6	0	10,000	0.0012	0.0040	0.0600
0.02	3	0	3,000	0.0006	0.0030	0.0300
0.06	1.5	0	1,000	0.0003	0.0020	0.0200
0.2	0.7	0	300	0.0002	0.0015	0.0150
1	0.3	55	60(50)	0.0	0.0001	0.0010
2	0.2	110 <sup>5</sup>	30(25)	0.0	0.0001	0.0010
10	0.1	110 <sup>5</sup>	6(5)	0.0	0.0	0.0005
100	0.03	110 <sup>5</sup>	0.6(0.5)	0.0	0.0	0.0

1. Resolution is defined as the typical 10 VDC range rms noise.
2. Normal mode rejection for power-line frequency +/- 0.1%.
3. Maximum rate for DCV, DCI, and 2-wire resistance functions (using zero settling delay, autozero off, etc.).
4. Autozero on for => 1 NLPC. Basic DC accuracy specifications include rms noise at 100 NPLC.
5. For <100 NPLC add appropriate "RMS Noise Adder" to basic accuracy specification.
6. For power-line frequency +/- 1% 75 dB for +/- 3% 55 dB. Only for 34411A.

Figure 2. Performance versus integration time for the Keysight 34410A DMM

for countries using 60 Hz AC line frequencies), we see that the spec is  $\pm(0.0015\% \times \text{reading} + 0.0004\% \times \text{range})$ . From Figure 2, we also see that on the 10 V range, and for integration times of less than 100 PLC, we must add 0.0003% of range to the accuracy to account for DC noise errors. Suppose that we wanted to read a voltage that was previously calibrated, traceable to NIST standards, to be exactly 5.000000 V and we wanted to measure this on the 10 V range at an

PLC of 0.06 (1 ms). We would expect the accuracy to be  $\pm(5.0 \times 0.000015 + 10 \times 0.000004 + 10 \times 0.000003)$ , or  $\pm 145 \mu\text{V}$ , and thus we would expect the DMM to display a steady reading somewhere between 4.999855 and 5.000145 volts, depending on how accurately we calibrated it. Since there is some unspecified base level of DC noise already included in the accuracy specification and we added 30  $\mu\text{V}$  of additional noise to it, we would expect to see something over  $\pm 30 \mu\text{V}$  of variation in the readings

due solely to DC noise. The peak-to-peak noise is thus something over 60  $\mu\text{V}$ . As this paper will show, this represents an rms noise of approximately 60  $\mu\text{V}/6$ , or 10  $\mu\text{V}$ . In fact, the DC noise specification is 15  $\mu\text{Vrms}$  on the 10 V range, so we would infer that 5  $\mu\text{Vrms}$  was the unspecified DC noise that had been included in the accuracy specification.

Analog-to-digital converter performance accounts for all DC noise errors. The second column in Figure 2 shows the DC noise specifications for this DMM. Note that it varies in relation to the number of PLCs that are chosen. For example, if  $NPLC = .06$  (.001 second, or 1000 readings/s), the A/D converter in this DMM has an error of  $\pm 1.5$  ppm rms x range. On the 10 V range, this is  $\pm 15$   $\mu$ Vrms.

Ultimately, we want to know how many digits of resolution we can get for a given integration time. Taking the 10 V range again as an example, we would expect to see a variation from reading to reading of  $\pm 15$   $\mu$ V. Suppose our DMM was calibrated such that it read exactly 5.000000. We would expect readings to range from 4.999985 to 5.000015 most of the time. It is plain that the 1  $\mu$ V digit is useless in this case, so we would actually expect the readings to be displayed as 4.99998 to 5.00002.

This is a 6-digit reading. Since we are on the 10 V scale and this DMM has 20% overrange capability, readings up to  $\pm 12.00000$  can be taken. The most significant digit can only go from 0 to 1, so it is called a  $1/2$  digit. Thus, the DMM is said to be a  $6\frac{1}{2}$ -digit DMM at this reading rate, one in which the last (10  $\mu$ V) digit can be expected to change from reading to reading by  $\pm 2$  counts most of the time. ("Most of the time" means 68.3% of the time, which is the definition of rms.) In reality, the actual performance has been measured to be better than the spec, and we would normally see only a  $\pm 1$  count change in the last digit 68.3% of the time.

The way we measure the DC noise performance is by applying a DC voltage and taking a few hundred sample readings. Since we also wish to eliminate any errors in a source, it is simplest to just short the input terminals (which gives us a 0 V input) and ignore nonlinearities that might cause some additional errors at higher input-voltage

settings. We then get a distribution in the readings that follows a statistical model. If the samples were distributed equally, we would call it a uniform distribution. A classic paper on quantization error written in 1948 by Bell Labs<sup>1</sup> shows that the act of quantizing an analog signal results in a uniform distribution, so if our measured readings were distributed in this way, we would say that the DMM readings were quantization-limited. In other words, the act of converting the analog value to digital would be the primary source of the error. If the samples were distributed in a Gaussian form (see Figure 3), the noise would be said to be caused by 1/f noise, thermal noise, nonlinearities and other analog errors. The manner in which we would compute the rms value is different for the two cases.

1. W.R. Bennett, "Spectra of Quantized Signals," Bell System Technical Journal, Vol. 27, July 1948, p. 446-471.

The Bell Labs paper on quantization error shows us that the rms noise is given by:

$$\text{rms noise} = \text{ideal resolution} / \sqrt{12}$$

where ideal resolution = voltage span /  $2^n$   
 (n = number of bits in the A/D)

For example, an 8-bit A/D that can measure voltages from -1 to +1 has an ideal resolution of  $(1 - (-1)) / 256 = 7.8 \text{ mV}$ , and we would expect to measure an rms noise of 2.3 mV, *not including any other analog errors due to circuitry between the input terminals and the A/D converter.*

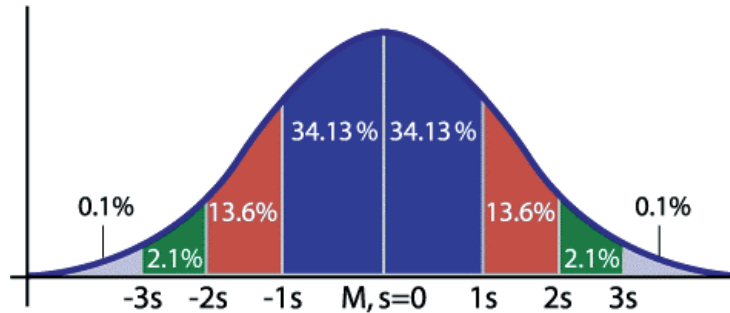


Figure 3. A Gaussian distribution

In most modern DMMs, the A/D converters have enough bits of native resolution that this quantization error is much smaller than other errors and can be ignored.

Since the other sources of error are distributed in Gaussian form, we can use the formula for standard deviation to compute the rms value.

### A quick refresher on standard deviation

Suppose we are given a population  $x_1, \dots, x_N$  of values (which are real numbers). The arithmetic mean of this population is defined as

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i = \frac{x_1 + x_2 + \dots + x_N}{N}$$

and the standard deviation of this population is defined as

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

Given only a sample of values  $x_1, \dots, x_N$  from some larger population, many authors define the sample standard deviation by

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

Since we are using a 0 V input signal, the mean is defined to be 0, so we can use the calculated sample standard deviation as the value of rms error.

$$\begin{aligned} \text{rms} &= \sqrt{\bar{x}^2 + s^2} \\ \bar{x} &= 0 \\ \text{rms} &= \sqrt{s^2} = s \end{aligned}$$

A Visual Studio.NET program was written to control a 34410A DMM using Keysight's T&M Toolkit software to make the graphing job and the instrument control very easy. The inputs were shorted and 1000 samples taken on the 10 V range with an aperture time of 1 ms. The data were plotted as raw data and as a histogram, as shown in Figure 4. Although quantization "buckets" are clearly visible in the graph on the right, the overall pattern is that of a Gaussian distribution. This DMM is therefore not quantization-limited.

In this case, 8.2  $\mu\text{Vrms}$ , or 45.5  $\mu\text{Vp-p}$  ( $\pm 22.8 \mu\text{V}$ ) of DC noise was measured. (Remember that the spec was 15  $\mu\text{Vrms}$ , so it is well within this limit.) Thus, there is a 68.3% probability that the measured value will be within  $\pm 8.2 \mu\text{V}$  of the actual value. A peak-to-peak value can be called three things that all have the same meaning:  $\pm 3$  sigma, six-sigma, or 99.7% probability. Thus there's a 99.7% probability that the measured value will be within  $\pm 22.8 \mu\text{V}$  of the measured value. Since these measurements were taken on the 10 V range, the reading would best be displayed as 0.00000, and we might expect to see it change from -0.00001 to +0.00001 once in a while and go as high as  $\pm 0.00002$  on a less-frequent basis.

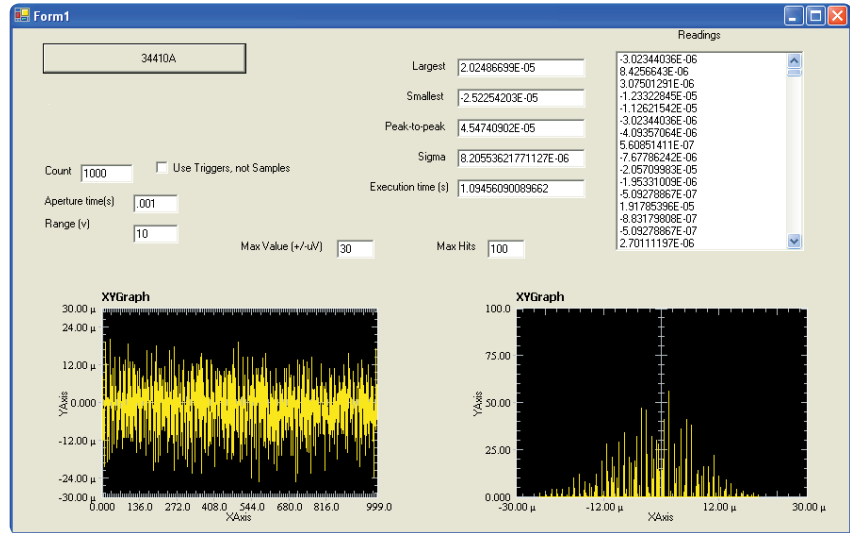


Figure 4. DC noise for the Keysight 34410A DMM

An effective number of digits (ENOD) can be calculated as follows:

$$\text{ENOD} = \log_{10} (\text{positiveSpan} / \text{rmsNoise})$$

for a 68.3% probability ( $\pm 1$  sigma)

or

$$\text{ENOD} = \log_{10} (\text{totalSpan} / \text{ppNoise})$$

for a 99.7% probability ( $\pm 3$  sigma)

Companies do not usually publish the internal span of their A/D converters, so it may not be possible to derive this number when you compare specs from one company to another. In the case of the 34410A, the internal range is  $\pm 14 \text{ V}$  on the 10 V range, even though the usable range is artificially limited to  $\pm 12 \text{ V}$ . So, plugging in,

$$\text{ENOD} = \log_{10} (14 / .0000082) = 6.2 \text{ digits (68.3\% probability)}$$

or

$$\text{ENOD} = \log_{10} (28 / .0000455) = 5.8 \text{ digits (99.7\% probability)}$$

Since the spec is based on rms noise, not peak-to-peak noise, we would call this a 6<sup>1</sup>/<sub>2</sub>-digit DMM on the 10 V range at 1000 readings/s. The same method can be used to calculate the ENOD for other ranges and speeds.

One can also use signal-to-noise + distortion ratio (SINAD) to calculate ENOD. There is a well-known formula relating signal, noise, harmonic distortion and effective number of bits in the A/D:

$$N(\text{effective}) = (S/(N + D) - 1.8)/6.02$$

Where  $S/(N+D)$  is the ratio of the signal power of a full-scale input to the total power of noise plus distortion, expressed in dB<sup>2</sup>. This formula reinforces the notion that noise errors come from many sources, not merely quantization error. A full discussion of this method of measurement is beyond the scope of this paper.

2. David A. Czenkusch, "High Resolution Digitizing Techniques with an Integrating Digital Multimeter", Hewlett Packard Journal, April, 1989, Vol. 40, Nr. 2, pp. 39-49.

## Conclusion

Establishing the number of usable digits for a DMM depends upon its calibrated accuracy and the performance of its A/D converter. In most modern DMMs, the quantization error due to analog-to-digital conversion is far less than that of other analog errors and can be ignored.

The effective number of digits is based on rms noise performance, which is a Gaussian distribution of noise that is generated inside the DMM by thermal noise, 1/f noise, harmonic distortion and other analog errors.

## Related Keysight literature

Product Overview  
5989-3738EN  
*Keysight 34410A and  
34411A Multimeters*  
[http://literature.cdn.keysight.com/  
litweb/pdf/5989-3738EN.pdf](http://literature.cdn.keysight.com/litweb/pdf/5989-3738EN.pdf)

Application Note  
5989-4279ENUS  
*8 Hints for Making  
Better Digital Multimeter Measurements*  
[http://literature.cdn.keysight.com/  
litweb/pdf/5989-4279ENUS.pdf](http://literature.cdn.keysight.com/litweb/pdf/5989-4279ENUS.pdf)

## Evolving Since 1939

Our unique combination of hardware, software, services, and people can help you reach your next breakthrough. We are unlocking the future of technology.

From Hewlett-Packard to Agilent to Keysight.



For more information on Keysight Technologies' products, applications or services, please contact your local Keysight office. The complete list is available at:

[www.keysight.com/find/contactus](http://www.keysight.com/find/contactus)

### Americas

Canada	(877) 894 4414
Brazil	55 11 3351 7010
Mexico	001 800 254 2440
United States	(800) 829 4444

### Asia Pacific

Australia	1 800 629 485
China	800 810 0189
Hong Kong	800 938 693
India	1 800 11 2626
Japan	0120 (421) 345
Korea	080 769 0800
Malaysia	1 800 888 848
Singapore	1 800 375 8100
Taiwan	0800 047 866
Other AP Countries	(65) 6375 8100

### Europe & Middle East

Austria	0800 001122
Belgium	0800 58580
Finland	0800 523252
France	0805 980333
Germany	0800 6270999
Ireland	1800 832700
Israel	1 809 343051
Italy	800 599100
Luxembourg	+32 800 58580
Netherlands	0800 0233200
Russia	8800 5009286
Spain	800 000154
Sweden	0200 882255
Switzerland	0800 805353
	Opt. 1 (DE)
	Opt. 2 (FR)
	Opt. 3 (IT)
United Kingdom	0800 0260637

For other unlisted countries:

[www.keysight.com/find/contactus](http://www.keysight.com/find/contactus)  
(BP-9-7-17)



[www.keysight.com/go/quality](http://www.keysight.com/go/quality)  
Keysight Technologies, Inc.  
DEKRA Certified ISO 9001:2015  
Quality Management System

### myKeysight

#### myKeysight

[www.keysight.com/find/mykeysight](http://www.keysight.com/find/mykeysight)

A personalized view into the information most relevant to you.

[http://www.keysight.com/find/emt\\_product\\_registration](http://www.keysight.com/find/emt_product_registration)

Register your products to get up-to-date product information and find warranty information.

### KEYSIGHT SERVICES

Accelerate Technology Adoption.  
Lower costs.

#### Keysight Services

[www.keysight.com/find/service](http://www.keysight.com/find/service)

Keysight Services can help from acquisition to renewal across your instrument's lifecycle. Our comprehensive service offerings—one-stop calibration, repair, asset management, technology refresh, consulting, training and more—helps you improve product quality and lower costs.



#### Keysight Assurance Plans

[www.keysight.com/find/AssurancePlans](http://www.keysight.com/find/AssurancePlans)

Up to ten years of protection and no budgetary surprises to ensure your instruments are operating to specification, so you can rely on accurate measurements.

#### Keysight Channel Partners

[www.keysight.com/find/channelpartners](http://www.keysight.com/find/channelpartners)

Get the best of both worlds: Keysight's measurement expertise and product breadth, combined with channel partner convenience.

